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ABSTRACT

An approach to teaching problem-solving based on using the computer software Mathematica is applied to the study of electrostatics and is compared with the normal approach to the module. Learning outcomes for both approaches were not significantly different. The experimental course successfully addressed a number of misconceptions. Students in the experimental course found the visualization tools to be more instructive than the problem-solving tools. This course imposed a high cognitive load on the students. This paper discusses how the course can be improved and how it can be a valuable supplement to the usual teaching approach. (Contains 20 references.) (DDR)

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Physics learning with a computer algebra system

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Contents

1. INTRODUCTION 1
2. THE LEARNING PROCESS 2
3. DESIGN OF A LEARNING ENVIRONMENT FOR PHYSICS PROBLEM-SOLVING 3
   - *3.1 The standard electrostatics curriculum* *4*
   - *3.2 Goals and requirements for improved instruction* *4*
   - *3.3 Types of instructional tools available* *6*
   - *3.4 Implementation of the learning environment* *7*
4. TRAINING FOR ENHANCED PROBLEM REPRESENTATIONS. A CLASSROOM EVALUATION 11
   - *4.1 Method* *11*
   - *4.2 Results* *13*
   - *4.3 Discussion* *18*
5. CONCLUSIONS AND RECOMMENDATIONS 21

APPENDIX A REVIEW OF SOFTWARE 23
APPENDIX B SAMPLE FROM THE EXPERIMENTAL COURSE 26
APPENDIX C POSSIBLE IMPROVEMENTS TO THE COURSE 31

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Physics learning with a computer algebra system

Towards a learning environment that promotes enhanced problem representations

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To become proficient problem-solvers, physics students need to form a coherent and flexible understanding of problem situations they are confronted with. This is important both for solving problems and for interpreting solutions. Still, many students have only a limited representation of the problems they are working on. Therefore, we devised an instructional approach to promote students' understanding of these problems, and to support them in forming associations between problem features and solution methods. The approach is based on using the computer algebra software *Mathematica* as a tool for problem-solving and visualisation. We implemented our approach in an electrostatics course module, and, in an experiment, we compared this module with the normal approach. Learning outcomes for both approaches were not significantly different. The experimental course, however, could successfully address a number of misconceptions. The experimental course students found the visualisation tools to be more instructive than the problem-solving tools. The experimental course was found to impose a high cognitive load on the students. Based on these outcomes, we propose how the course could be improved. We conclude that the experimental course could, with some improvements, be a valuable supplement to the usual teaching approach.

1. Introduction

In previous studies we demonstrated that mental representations and reasoning processes in weak problem solvers are qualitatively different from those in proficient problem solvers (Savelsbergh, De Jong, & Ferguson-Hessler, in press; 1997). Among the most salient differences we found were: the lack of coherence in the weak problem-solvers' problem representations, the sparseness of their problem representations, and their lack of associations between problem features and solution methods compared to proficient problem solvers. We argued that these features of weak problem solvers' problem representations form a serious impediment to successful problem-solving. Therefore, here

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* Paper to be presented at the NARST annual meeting 1998 in San Diego, CA. The experiment described in this paper was conducted at the Faculty of Physics and Astronomy, University of Utrecht, the Netherlands. The following members of the staff contributed substantially to the successful implementation of the experiment (alphabetical order): Cees Alderliesten, Rudy Borkus, Ger Engelbertink, and Jan Kuperus. We also thank Frans Blom (Eindhoven University of Technology) for commenting on early versions of the testing materials. Correspondence should be addressed to the first author. Postal address: Eindhoven University of Technology, Faculty of Technology Management, PO Box 513, 5600 MB Eindhoven, The Netherlands. E-mail: E.R.Savelsbergh@tm.tue.nl.

we present an instructional format we devised to promote the students' understanding of problems, and to support the formation of associations between problem representations and solution information.

In this paper, we start by reviewing the relevant learning processes. Next, we discuss the possible contributions of several types of learning tools, leading to the choice we made for a particular type of tool. Next, we present the implementation of a course module based on this tool, and an experiment we conducted to test the effectiveness of the module. We end with a discussion of design improvements and design guidelines that were suggested by the outcomes of the experiment.

2. The learning process

Learning how to solve a particular class of physics problems fluently, is a complex and time-consuming process. As a primer, a student may listen to a lecture, or interact with a computer simulation to become acquainted with the domain concepts. It is only after this first encounter, however, that the student begins to learn how to solve problems. The continued learning process first requires the learner to combine information from different sources, such as the textbook, previous problem-solving experiences, and mathematics and physics pre-knowledge. Second, it requires the learner to go beyond the literal information presented in order to create understanding, to see implicit regularities, and to learn to routinely apply domain theories. It is common that impasses and misunderstandings arise during the process, and insight often comes only after a period of 'wandering in the dark'. After the initial conceptual barriers have been overcome, it still requires considerable practice to become fluent in selecting the right solution step in a particular circumstance, in recovering from errors, and in carrying out the selected solution steps.

This sequence of stages in the learning process parallels the different phases in a child's development as defined by Piaget (1970). Van Hiele (1957; 1986) was the first to recognise that one passes through similar phases each time one starts learning a new subject. Van Hiele distinguishes between a visual level, a descriptive level, and a theoretical level of understanding, and he continues with more abstract levels beyond these (Van Hiele, 1986, p. 53). According to Van Hiele's theory, the levels follow in a strict order, and specific materials and specific kinds of problems are associated with each level. The attainment of a following level of understanding requires a specific focus and specific activities. To begin with, attainment of the visual level requires extensive playing with the relevant concrete objects. In this stage of learning, the students have no means to formulate goals yet, and consequently they do not yet involve in problem-solving. To attain the second level the student has to practice in expressing the system's properties in words. In attaining the third level, practising with proof problems has a role. Quite sometimes students can learn tricks to solve problems beyond the level they have mastered without them having understood these problems. According to Van Hiele, this does not contribute to understanding, as a transition from one level to the next is to be triggered by the crisis that occurs when the lower level approaches no longer suffice.

Van Hiele's theory is primarily grounded in high school geometry, which might explain the emphasis on playing with concrete objects. In physics at the university level, the role of concrete objects and representations is different from that in geometry because often the concrete representations are crude approximations to illustrate theoretical ideas. Moreover, unlike high school students, university students have some background in reasoning with mathematical concepts already. Therefore, the strict temporal ordering proposed by Van Hiele cannot be directly transplanted to university physics. Still, also in

university physics we can distinguish the levels described above, and we follow Van Hiele in that understanding at a higher level needs to be based on understanding at the lower levels.

The theory by Van Hiele outlined above clearly illustrates the constructivist view on learning. Constructivist approaches emphasise that knowledge is constructed by students themselves rather than being transmitted from teacher to student. Many constructivist theories, such as Van Hiele's but also Vygotsky's, and Bruner's, take a global developmental or cultural perspective. The construction of knowledge can also be analysed at a more detailed cognitive level in terms of information processing, however.

From an information processing point of view, there are two relevant approaches to the learning process described earlier: one is the broad class of production-rule theories; the other is the schema theoretic approach. First, we look at production-rule systems. We take Anderson's ACT* theory as a well-known instance of the production-rule approach. ACT* theory distinguishes three major phases in learning: acquisition of declarative knowledge, proceduralisation of the knowledge and, finally, tuning of the skills acquired (Anderson, 1983). These steps are worked out in detail in ACT* and, based on this detailed theory, some quite specific tutoring principles have been proposed (Corbett & Anderson, 1992). The most notable recommendations are: to emphasise the analysis of goals and sub-goals, to provide instruction in the problem-solving context, to provide immediate feedback on failures, to provide ample opportunity for practice and, finally, to minimise working memory load.

The other view comes from schema theory, with its focus on the formation of networks and structures (Rumelhart & Norman, 1981, 1981.). Schema theories have stressed the organisation of knowledge in clusters, with each cluster centred on a type of problem that requires a particular approach. The learning process in this view is characterised as failure-driven and explanation-based. Although the approach is less clearly associated with a particular person or named theory, the approach also leads to some clear recommendations for instruction. The most relevant are: to explicate the expert's reasoning process; to present a realistic problem-solving context where the learner has to construct a situation model; and, finally, to put the learner in control of the learning process, which includes the diagnosis of failures (Glaser & Bassok, 1989).

Though both approaches account for the importance of content-related abilities, the schema theoretical view quite naturally seems to account for impasses, sudden insights, and the importance of situation representations. ACT*, in contrast, depicts learning as a smooth process, resulting in habits. Consequently, the learning processes that occur in complex problem-solving domains may be better characterised by a schema theoretical description than by rule-based approaches such as Anderson's (1983). Therefore, in cases where both theories lead to contradictory advice we might best follow the advice from schema theory.

3. Design of a learning environment for physics problem-solving

In this section, we discuss the choice of a particular type of learning support and the subsequent implementation of a course module. We first describe the course as it has been taught for many years, with its goals, its approach and its shortcomings; then we discuss the possible improvements to the course; we review the merits of available tools; we explain our choice for a particular type of tool; and, finally, we discuss the implementation of an adapted course module that is based on this tool.

3.1 The standard electrostatics curriculum

The domain of our study is an electrostatics course module, taken from the common curriculum for first-year physics students. The module is taken as part of a longer course on electrodynamics. Topics covered in this module are: charge distributions, symmetries, Coulomb's law, Gauss' law, dipoles, multipoles, conductors, computation of potentials with given boundary conditions, dielectrics and polarisation.

The course has three major components: lectures, work groups and homework. Lectures last for two hours, and a typical audience is about one hundred students. In the lectures, the theory is presented and examples of typical problems are worked out. During work groups, small groups and individual students are assigned a set of problems to solve. A tutor is available for every twenty students who assists if problems arise. Students are expected to solve additional problems at home, and to study the book 'Introduction to Electrodynamics' (Griffiths, 1987). The projected total workload for the course is 80 hours for the average student.

The main aim of the course is to teach thorough understanding of fundamental concepts and approaches. Here, the groundwork is laid both for more advanced theoretical courses and for application-oriented technical courses. The concepts taught are discussed in a simplified way, and the methods presented are only practical for some idealised problems. However, because later courses build on the material, the student should become fluent with the basic concepts, relations between them, applicability of the methods and assumptions underlying them.

It is a well-known problem that students, even though they have learned the methods, fail to see how and when these methods could be applied in new situations. More specifically, in this course, students fail to see how they can use the geometrical properties of a situation to simplify the problem. This might be explained from our earlier work where we have demonstrated that novices in this field do not integrate solution information in their mental problem representations (Savelsbergh et al., submitted).

Another problem is that weak students frequently seem to misunderstand problem descriptions, and often fail to make proper drawings of situations (Feiner-Valkier, 1997). This might be explained by the students inability to form a coherent understanding of the problems (Savelsbergh et al., submitted; 1997), and by the weak students failing to switch between propositional and pictural representations (De Jong and Ferguson-Hessler, 1991).

3.2 Goals and requirements for improved instruction

Based on our view of the learning process and on shortcomings of the current approach, which we have outlined in the previous sections, we assume that students could benefit from a training procedure promoting the construction of problem representations. Our primary goal is to enrich students' mental models of problem situations with elements helping them to construct an integrated model of the situation, and to connect this situation representation to solution information.

The constructivist viewpoint suggests that an effective training procedure is best accomplished by helping students to construct elaborate problem representations themselves, rather than directly providing them with extensive problem descriptions. Both schema theory and production rule theories suggest that good problem representations can only be learned in the context of real problem-solving activity. Therefore, our approach is to support the formation of problem representations during practice problem-solving. In addition it may be helpful to already start with the integration of theory and situations while studying theory and examples.

Previous studies suggest that proficient and weak students need qualitatively different kinds of support. Strong evidence comes from our own research where we found that proficient students, in contrast to weak students, perform better on a problem sorting task with expanded problem descriptions than with minimal descriptions (Savelsbergh et al., 1997). The difference could be interpreted as indicating that weak students are still wrestling to attain the first Van Hiele level, whereas proficient students are already working towards the second or third level of understanding. This gives rise to the supposition that whereas weak students should primarily be supported in constructing *coherent* problem representations, proficient students may benefit more from support in constructing *flexible* problem representations. The different needs of different students can be met either by a system based on an advanced learner model that offers customised instructions, or by an open system that lets students decide what kind of support they need themselves.

Students may be helped in constructing coherent problem representations by stressing the relation between the propositional problem description and the visual representation of the problem because the visual format reveals relations between propositions. Flexible coordination of situation representations and solution procedures may be stimulated by making solution procedures recognisable as functional blocks and by demonstrating how modifications in the situation affect the use of procedures.

If students are to be trained in constructing problem representations themselves, they should be allowed to elaborate on their knowledge of the situation freely. So they must be free to modify the problem or pose their own problem. Therefore, the teacher should be reticent with judgements, in order not to frustrate the exploratory process. In other words: the students should be 'owners' of the problem. These requirements suggest a learning environment that allows the learner much freedom. Consequently, such a learning environment will call upon the regulative abilities of the student. Students who lack these abilities may easily loose their way. Moreover, a strong appeal to the student's regulative abilities may interfere with content learning, because of cognitive overload. To reduce these threats, some guidance should be provided and care should be taken to minimise cognitive load extraneous to the task. This can be done in several ways: Firstly, all required information can be integrated in a single structure (Chandler & Sweller, 1991; 1992). Secondly, worked examples can be presented instead of or in addition to problem-solving tasks (Sweller and Cooper, 1985; Zhu & Simon, 1987; Paas & Van Merrienboer, 1994). Finally, 'goal-free problem-solving' (i.e. computing anything you can for a given situation) also helps to reduce cognitive load (Sweller, 1988[E.R.1]). The latter two recommendations are clearly at odds with ACT* and schema theory recommendations and findings (Glaser & Bassok, 1989; Corbett & Anderson, 1992). The differences might be explained by differences in learning tasks and desired types of learning outcomes. Still, worked examples could be used as an introduction to problem-solving tasks.

The requirements we have discussed thus far, and especially the need for visualising solutions, suggest the relevance of a computer-mediated learning tool. This would impose an extra source of cognitive load, namely the interaction with the computer itself. Since students differ greatly in the amount of computer experience they have, this will pose more problems for some than for others. Therefore, it will be important to offer appropriate user support. A further factor that may affect learning in a computer-mediated environment is computer anxiety. On the one hand, it has been claimed that there is no relation between computer anxiety and learning results when outcomes are corrected for computer experience (Szajna & Mackay, 1995). On the other hand there is clear evidence that anxiety affects information processing (cf. Wilder & Shapiro, 1989; Baron, Inman, Kao, & Logan, 1992). So, it may be worthwhile to attempt reducing computer anxiety, if it were only to improve the students' motivation.

3.3 Types of instructional tools available

In the previous section we identified two major problems: weak students' lack of internal structure in their problem representations, and weak students' failure to switch between different representations. These problems lead us to search for an instructional tool that would assist students in actively constructing problem representations themselves, rather than providing them with ready-made situation models. Moreover, we were looking for a system that would give some form of feedback on representations created by the student. Therefore, our focus was on computer-based tools. In Appendix A, we briefly present a representative sample of software packages and discuss the role these packages could play as instructional tools. The many computer programs that are made for demonstrating one particular situation – like most of those from the American Institute of Physics – have been excluded from that review. We restrict our discussion to software that can be used for the study of a broader range of physical situations. The programs that we examined fall into four broad categories classes:1 visual interface numerical simulation environments, simulation packages with combined visual and formula interface, numerical packages with formula input and visualisation facilities, and computer algebra packages. Below we will briefly discuss the four.

Visual interface numerical simulation environments

In visual interface numerical simulations (such as XYZet, Interactive Physics, and SimQuest), the student can interact with a numerical simulation of a physics phenomenon. The user can construct a situation by clicking and dragging elements, vary quantities by setting sliders etcetera. These packages are appropriate for constructing situation models. They do not address the propositional representation, including the use of formulas, however. Moreover, they cannot be used in formal problem-solving. In summary these packages may be better suited to use for instruction at the first Van Hiele level, i.e. to gain an intuitive understanding of the concepts used.

Simulation packages with combined visual and formula interface

Numerical simulation packages with combined visual and formula interface (such as Modellus and Labview) support the learner in switching between visual and propositional representations. The user specifies a physical system using formulas, and then specifies a visualisation. These packages do support the construction of situation representations, they force the user to specify the situation precisely. Moreover this type of environment is unrestrictive. However, it does not support formal problem-solving.

Numerical packages with formula input and visualisation facilities

Numerical problem-solving tools with visualisation facilities (such as MathCad and MatLab), can be used to specify and visualise situations and solutions. They can also be used to practise problem-solving, but the problem-solving methods they can be used for are restricted to numerical methods. An advantage of numerical methods is that they can be used to solve more complex problems. A disadvantage of these methods is that they are farther from the theoretical framework of the domain, and, since the outcomes are expressed as numbers rather than formulas, the outcomes do not provide useful insights.

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[1] Support tools could also be classified with respect to the van Hiele level of cognitive development they address. XYZet and Modellus could be said to address the first van Hiele level (concepts are still inarticulate, naive thinking). Mathematica could be supposed to address the second or the third van Hiele level (the learner is able to apply operative properties of a well-known concept, and the learner is able to prove properties respectively).

Computer algebra packages

Computer algebra packages (such as Derive, Maple and Mathematica) are similar to those in the previous group, but they can also be used to solve symbolic equations.

Comparison of the above packages leads to the conclusion that computer algebra packages offer the best functionality for supporting the learning processes we are considering. A computer algebra system (CAS) in itself is no more than a high level programming language for symbolic and numerical computation. In general, a CAS has both imperative and procedural programming facilities. For our intended use, three properties of CASs are of importance: firstly, CASs demand precise specification of problems, in a highly constrained formal specification language; secondly, CASs take over algebraic calculations; and finally, most CASs have visualisation facilities. The required precise specification of the problem could direct the student's attention to the properties of the problem situation. Assistance in algebraic calculations may help students to focus on the main line of the solution rather than algebraic calculation details. Moreover, once a first case has been worked out, it becomes easier to study the effect of changing situation properties. Finally, visualisation facilities may help the student to gain a better understanding of solutions by drawing graphs of formulas that otherwise would remain abstract.

3.4 Implementation of the learning environment

We decided to implement our learning environment using a computer algebra system as a problem-solving and visualisation tool. Among the more well-known examples of these systems are Mathematica and Maple. Because the students who where intended participants in the study already had some experience in working with a particular CAS, namely Mathematica, we decided to use that program.

Mathematica (version 3.0), like Maple, has an advanced hypertext interface that can present pictures, and conventionally formatted mathematical formulae alongside the user-typed input. So, a course module written in Mathematica can be set up as an 'interactive book', with all the necessary information – including theories – presented on-screen. Compared to a regular hypertext, the major differences are that students can modify the content, that the package supports problem-solving and that the package can be used as a visualisation tool. As discussed in the previous section, these features of CASs could be used to support the learning process. To stimulate the use of computation and visualisation facilities, we set up assignments to require the use of these features.

A CAS can be used in many ways. We choose to provide an integrated learning environment, that would present the theory in brief, followed by worked examples, after which various types of assignments would follow. Theory would be presented only briefly, because for extensive reading a book is more convenient. The first assignments on a topic would be highly structured, requiring the learner to modify something in a worked example or to complete an incomplete solution. Later assignments would be more open, requiring the learner to construct the entire solution. This set-up was intended to minimise extraneous cognitive load.

The worked examples help to reduce the effort that goes into mastering the programming language. The structured assignments are intended to focus on variations of situation properties within a class of problems, and to the consequences the properties have for the solution of the problem. Thus, structured assignments may help the students identifying relevant properties of a situation (=elaboration). The visualisation assignments may help to connect a concrete (visual) physical representation to the abstract formalism of both the situation and the solution (=visual elaboration). Finally, practice problems require the

problem solver to elaborate on the problem statement (visual and propositional) and, moreover, they provide a training opportunity. The difference from normal practice problems is the support offered by the CAS.

The experimental course was intended to represent 8 to 10 hours of workload on the average student. The general subject of the course was 'special techniques for calculating potentials'. There were four sections:

1. general introduction and instruction Mathematica
2. introduction E-field and potential
3. image charges
4. dipole and multipole expansion.

In Figure 1 we present a brief example of a topic in the experimental course (a longer sample can be found in Appendix B). The example in Figure 1 begins with a brief summary of the relevant theory, followed by a worked example. In the worked example here the text in lines labelled In is already present when a student starts working with the material. When the student executes the commands in these lines, by pressing SHIFT-ENTER, the computer generated output labelled Out appears. The student can also modify the input lines and then execute the commands again to examine the effects of a different situation. Below the worked example follow two examples of – parts of – structured assignments. The input shown in these examples (in[13] and in[14]) is to be thought up and typed in by the students themselves. To facilitate entering symbols and expressions, a floating 'palette' was provided in a separate window. The students could pick an element from the palette by clicking it.

From a mathematical point of view our problem is to solve Poisson's equation in the region $z > 0$, with a single point charge q at [0, 0, d], subject to the boundary conditions:

1. V[x, y, 0] = 0 (since the conducting plane is grounded)
2. $\lim_{r \to \infty} V[r]=0$ with r the distance from the charge.

The first uniqueness theorem guarantees there is only one function which meets these requirements. If we can discover such a function, it must be the answer. We now replace the conductor with a charge q2 at [0, 0, z2]. For this configuration I can easily write down the potential:

Theory in brief

```
In[10]:= v[{x_, y_, z_}] := Monopole[q1, {0, 0, d}, {x, y, z}] + Monopole[q2, {0, 0, z2}, {x, y, z
```

We now apply the first boundary condition: we demand that V[x,y,0] = 0. This is done choosing V = 0 for some arbitrary points in the plane, and solving the resulting set of equations (there are two unknowns: q2 en z2, so, two equations are needed to solve the problem):

Worked example

```
In[11]:= res1 := Solve[{v[{0, 0, 0}] == 0, v[{1, 0, 0}] == 0}, {q2, z2}]
         res1

Out[11]= {{q2 → -q1, z2 → -d}, {q2 → -q1, z2 → d}}
```

expression input

Clearly, the problem has two solutions: one is the so called image charge: an opposite charge at distance d behind the plane. The other one is an opposite charge, but at the same place as the original one. This is a trivial solution: no field remains. Turning back to the original problem: we try the first solution, as it satisfies the first boundary condition and q1 is the only charge in the region of interest ($z > 0$), so:

```
In[12]:= vopl[{x_, y_, z_}] = v[{x, y, z}] /. res1[1]

Out[12]= -q1 / (4 π √(x^2 + y^2 + (-d - z)^2) ε0) + q1 / (4 π √(x^2 + y^2 + (d - z)^2) ε0)
```

▼ **Problem** Check whether the solution satisfies the second boundary condition as well

To check the second boundary condition, you might use the Limit operator

Structured assignment symbolic computation

```
In[13]:= Limit[vopl[{x, y, z}], x → ∞]

Out[13]= 0
```

▼ **Problem** Visualise the potential and the field in the region $z>0$

```
In[14]:= Plot3D[vopl[{x, 0, z}] /. {q1 → 1, d → 1}, {x, -2, 2}, {z, 0, 4}, PlotPoints → 30,
                PlotRange → {0, 3 * 10^10}, ClipFill -> None, ViewPoint -> {2.5, -0.8, 1.5}]
```

Structured assignment visualisation

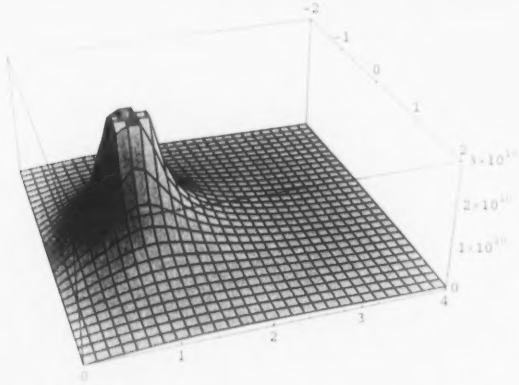

Figure 1 Brief example from the section on image charges in the experimental course(translated)

4. Training for enhanced problem representations, a classroom evaluation

The experimental course we had designed was supposed to improve students' understanding of physical situations and to strengthen the relation they see between solution methods and situation features. To test these hypotheses, we designed and carried out an experiment. In the present section we discuss the method and the outcomes of the experiment.

4.1 Method

We compared two conditions: a computer group where the experimental course module was used and a 'traditional' group. In the traditional group the students used paper and pencil to solve similar problems as the ones presented as open assignments in the computer course. Spread over three days a student had 7 hours available on the learning task, which is somewhat shorter than the projected time required for completing the experimental course module, so that also quick students could use all time available. Students in both groups worked under the supervision of a tutor. The composition of the experimental groups is discussed first. After that, we discuss the instruments we used for assessing learning outcomes. Finally, we discuss the instrument we used to assess the students' evaluation of their learning and of the course module.

4.1.1 Subjects

The experiment was conducted at the Faculty of Physics at Utrecht University in The Netherlands. There are approximately 90 first-year physics students. All these students were contacted individually and invited to participate. In total 42 students agreed to participate. These students were assigned to the two experimental groups by the experimenters. On the basis of available prior performance scores, two groups were formed of equal ability. Because of the limited number of computers available, the computer group had to be split in three equal sub-groups taking the course at different times. To maintain equal group sizes, the control group was also split in three sub-groups. In the first session 33 of the 42 students attended. (17 in the computer group and 16 in the traditional group). During the two subsequent sessions, all remaining 33 students kept attending. Students in both groups were paid *f*50 (US$ 25 approximately) after they had completed all three sessions.

4.1.2 Measures of the learning outcome

4.1.2.1 Examination score and average prior-performance

As a first measure of the learning outcome we have the scores on the regular final examination for the course. For this examination students had to do three assignments consisting of 14 sub-problems in total. In a typical assignment, the first sub-problem requires a basic understanding of the situation, the next sub-problems apply to standard solution procedure, and sometimes in the final sub-problem some more creative and subtle manipulations are required. The scores on the sub-problems could be used to compute a reliability parameter for the total examination score. These scores were also available for students not participating in the experiment.

To match the control group and the experimental group with respect to prior performance and to correct for differences in average performance level, we collected the students' scores on four prior tests. These prior tests were: the VWO (=grammar school level) final

examination scores for physics and for maths for the sciences, and the university final tests for introductory mechanics and introductory relativity theory. In earlier research we have demonstrated that these scores provide a reliable indicator of prior performance (Savelsbergh et al., submitted). The prior performance score was taken to be the mean of these four test scores. Missing values were replaced by the mean score for that test. The prior scores were also available for students who did not participate in the experiment, so that we compare the performance of the students in our sample to the overall population performance.

4.1.2.2 Pre and post-test

Because we were specifically interested in students' understanding of problems and in their choices of solution methods, we developed a pool of test items that were supposed to assess just these abilities. All items were tested by several expert teachers. The expert teachers also judged the quality of the distractor answer alternatives, and they gave a difficulty-rating to each item. As a final result we had a set of multiple-choice items of two different types: elaboration questions, and solution type questions. In the first type was asked for a property of the situation, in the second type was asked for the appropriate solution method for the problem at hand. Examples of the two types are given in Table 1.

Table 1 Examples of the two types of items that were used in the pre-test and post-test.

Elaboration item	**Solution-type item**
Take the following charge distribution What is correct for the field outside the charge distribution?	The electron charge distribution of a hydrogen atom can be described by the following formula $\rho(r) = c.e^{-r/a}$. If you want to describe the electric field caused by this charge distribution, your best option is:
☐ the field equals zero nowhere ☐ the field equals zero in exactly one point ☐ the field equals zero in exactly two points ☐ the field equals zero on an entire surface in space	☐ to use Coulomb's law ☐ a multipole approximation ☐ to use Gauss' law ☐ to integrate over the spatial charge contributions explicitly

From the pool of test items we constructed a pre-test and a post-test. The item types, subjects addressed and difficulty of the items (as judged by the expert teachers) were adjusted to have equivalent tests. After final corrections, a pre-test of 21 items and a post-test of 24 items resulted.

4.1.3 Evaluation and observation

Apart from direct performance outcomes, we were interested in the students' opinions about the experimental course, and in the learning process that took place in the experimental group. To assess the students' opinions we constructed a questionnaire. Because of limited resources, our only way to keep track of the learning process in the experimental group was by informal notes kept by the experimenter who taught the experimental course.

We made two versions of the questionnaire, one for the experimental group and one for the control group. Both versions of the questionnaire addressed the following major issues:

- Number of assignments completed
- Attractiveness of the instruction *(3 items)*
- Difficulty of the subject *(2 items)*
- Navigation and control *(4 items)*
- Help and collaboration *(5 items)*

In the questionnaire for the experimental group, there were additional items to assess the use and appreciation of features of Mathematica, such as graphing and computing integrals, and instructional elements, such as worked examples and practice assignments. Except for the number-of-assignments-completed items, all items were to be answered on five point Likert scales.

4.2 Results

4.2.1 Learning results

We had chosen two measures for learning outcome: the examination result and the adapted test that, as we have discussed in the previous section, was supposed to assess more specifically the quality of the student's situation representations and the linking between situation representation and solution approach. From now on we will refer to the tests as regular test and adapted test respectively. Next, we will discuss the outcomes of both measures.

In total 56 subjects took the regular test. Except for one student from the computer group, students who had participated in the experiment also took the regular test. To assess the reliability of the test score, we computed Cronbach α, taking the 14 sub-problems in the test as items. With all 56 subjects included we found a reliability of $\alpha = .90$. Descriptives of the scores for groups of students are given in Table 2.

Table 2 Descriptives of the prior performance scale, the regular test scores, adapted pre-test and the adapted post-test.

		Regular tests		Adapted tests	
Condition	*n*	Prior results[a]	Electro-statics[a]	Pre-test[b]	Post-test[b]
Computer	17				
M		7.8	6.1[c]	0.53	0.49
SD		0.8	2.2[c]	0.12	0.15
Traditional	16				
M		7.4	5.8	0.52	0.52
SD		1.0	2.0	0.15	0.14
Non-participating	24[d]				
M		6.5	4.1		
SD		1.6	2.8		

[a] scores represent grades according to Dutch school system: 1 is very poor, 10 is excellent.
[b] scores represent proportion correct answers.
[c] $n = 16$ because one of the students did not take the final test.
[d] only students who did take the regular final test were included.

When we test the significance of the between-group difference, we have to include the prior performance level as a covariate, to correct for prior differences between the groups. The four-item prior-performance scale is a quite reliable estimator of the general prior

performance level, $\alpha = .87$, $n = 57$. After the inclusion of this covariate in the analysis, no significant effect of the experimental treatment remains, $F(1,29) = 0.26$, $p = .77$.

We also compared the results of participants in the experiment to those of the students who had not participated. Participants clearly differ from the non-participating students: both on prior performance and on the regular test their scores were significantly better ($F(1,55) = 8.17$, $p = .006$ and $F(1,54) = 10.81$, $p = .002$ respectively), which indicates that mainly proficient students volunteered. Although the participants in the experiment were not drawn randomly from the population, we can get some idea of the main effect of attending an extra 7 hours of instruction by comparing the final test scores between participants, and non-participants. Of course, the prior performance level has to be included as a covariate again to compensate for the general difference in achievement level. Surprisingly, the gain for the students who took the extra instruction, relative to the results of non-participating students, is not significant, $F(1,53) = 0.50$, $p = .48$.

Apart from the main effect of the treatment a second, and equally important, issue is whether the treatment reduces or increases the difference between weak and proficient students. Here again we found no significant interaction between experimental condition and general performance level, $F(1,28) = 0.70$, $p = .41$. The trend in the data, however, suggests that the experimental instruction is more favourable to weak students than to good students (Figure 2).

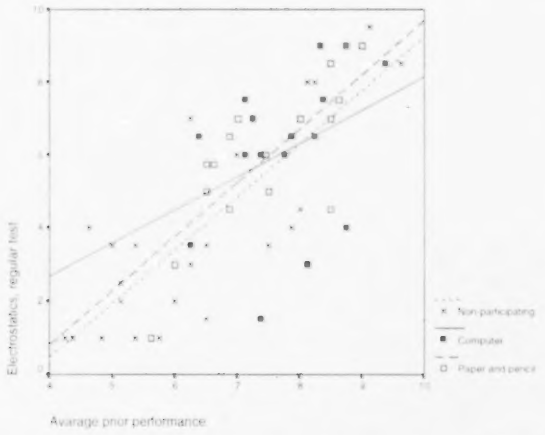

Figure 2 The scores on the regular final test, for both groups in the experiment and for the non-participating students, plotted as a function of their average prior-performance levels.

We now turn to the scores on the adapted test. Both a pre-test and a post-test were administered. For both the test reliability was estimated using Cronbach's α. For the pre-test, we found a quite low reliability, $\alpha = .42$, $n = 21$, and for the post-test we also found a poor reliability, $\alpha = .55$, $n = 24$. These values suggest that the items were not parallel tests of the same ability. The total number of participants was too low, however, to permit any sensible further analysis of the tests such as item response models or multidimensional scaling techniques, that could be used to find out whether all items respond to the same ability. Although the pre and post-test were intended to measure the same ability, the scores on both tests cannot be compared directly, as we have no empirical data on the relative difficulty of both tests. Therefore, we compared the scores of both groups on the post-test, and we included the pre-test score as a covariate. We tested both the main effect of experimental condition and the interaction between the experimental condition and the covariate, but neither gave a significant result

($F(1,29) = 1.92$, $p = .18$ for the main effect and $F(1,29) = 1.51$, $p = .23$ for the interaction). Here the main effect displays a trend in favour of the traditional group, and the trend in the interaction suggests that good students may be better off with the computer learning environment.

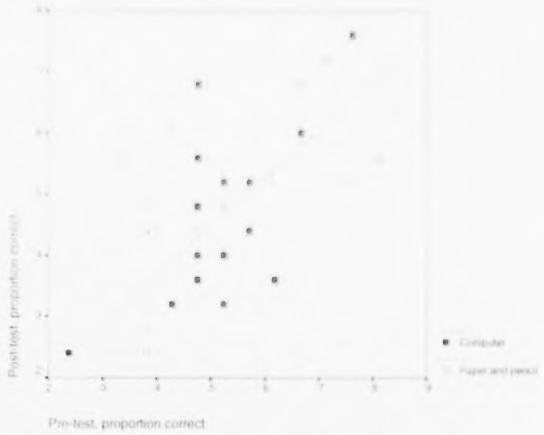

Figure 3 The scores on the adapted post-test, for both groups in the experiment, as a function of the scores on the pre-test.

4.2.2 Evaluation

To gain insight in the mechanisms that led to the learning outcomes, we also collected some evaluative information regarding the process. The evaluation consists of three parts: a set of questions posed to student in both groups, a set of questions specifically addressing features of the computer algebra environment and, finally, remarks collected from the evaluation forms.

First, we discuss the items that can be compared across groups. The first issue addressed was the number of assignments completed: students in both groups had to solve the same sets of assignments, the question was about how many of these assignments the student had completed. The worked examples and completion exercises that students in the computer group had to study prior to starting with open assignments were not included in the comparison. The average number of assignments completed was significantly lower for the students in the computer group, than it was for students in the traditional group, $F(1,31) = 29.1$, $p < .001$ ($M = 3.6$, $SD = 2.1$, and $M = 7.0$, $SD = 1.5$, respectively). This indicates that students in the experimental group spent much of their time on worked examples and completion exercises.

We found no evidence that students found one condition more attractive, $p = .89$. In the further items that could be compared across both groups we found a number of differences between both groups. The students in the Mathematica group judged the physics content in the experimental instruction to be simpler than the students in the control group did, $F(1,31) = 4.48$, $p = .042$. Because the difference was not reflected in the students' judgements about the difficulty of the general course, we must conclude that the difference is in the content of the course module we used in the experiment. This is easily understood because students who worked with the experimental course spent so much of their time on the introductory assignments. Given this difference, it is not quite surprising that the students in the control group score higher on the question whether they had learned much about physics content.

We had also posed some questions regarding regulation, navigation and confusion. Students in the computer group clearly had more trouble finding their way than students in the traditional group had, as is demonstrated by the scores displayed in Table 3 where the computer group had higher scores on all four items.

Table 3 Evaluation items related to navigation and confusion (the wording was slightly different for both groups: adaptations for the traditional group are italicised).

While I was working with the system *(solving the exercises)*, I often forgot what I did before, $F(1,31) = 3.39$, $p = .075$.
The system *(I)* usually could solve the problem, once I had entered *(formulated)* it correctly, $F(1,31) = 7.31$, $p = .011$.
While I was working with the system *(solving the exercises)*, I often lost overview of things that appeared on the screen *(I had already written)*, $F(1,31) = 10.343$, $p = .003$.
While I was working with the system *(solving the exercises)*, I had to look up things in earlier work often, $F(1,31) = 9.40$, $p = .004$.

We found marginally significant differences in the amount of help required and the type of help required. The computer students required slightly more help, $F(1,31) = 3.28$, $p = .08$, and the help they needed was less focussed on physics understanding, $F(1,31) = 3.07$, $p = .09$. Scores for the students in the Mathematica group, indicated that they needed more help on Mathematica, then they did on conceptual physics problems, $F(1,16) = 5.54$, $p = .03$.

The next part of the evaluation concerns specific features of the Mathematica learning environment. Several features were assessed on each of the following aspects: clarity, attractiveness, difficulty and instructiveness. Two of the items addressed inherent features of Mathematica itself, namely visualisation, and computation of gradients. For these features we also asked after the amount of use. Three other items addressed the elements that we had built into the learning environment: worked examples, completion tasks and open assignments. The results of the evaluation are summarised in Table 4 and Table 5.

Table 4 Evaluation outcomes for two features of Mathematica ($n = 17$). Scores are on 5-point Likert scales.

	Amount of use	Clarity	Attractiveness	Difficulty	Instructiveness
Visualisation					
M	4.06	4.53[a]	4.06	2.71	3.94
SD	0.97	0.62	0.97	1.05	0.75
Gradient					
M	3.71	3.87[a]	3.65	2.29	2.41
SD	1.21	0.96	0.79	1.05	1.12

[a] $n = 16$ because of one missing value.

Table 5 Evaluation outcomes for three elements of the learning environment ($n = 17$). Scores are on 5-point Likert scales.

	Clarity	Attractiveness	Difficulty	Instructiveness
Worked examples				
M	4.53	3.88	2.06	3.53
SD	0.80	0.70	0.56	1.33
Completion assignments				
M	3.88	3.59	3.12	3.53
SD	0.86	0.51	1.11	1.12
Open assignments				
M	3.12	3.53	3.88	3.82
SD	1.11	0.72	0.78	0.72

At first sight it is clear that, overall, *visualisation* is judged more favourably than *gradients*. We tested the significance of the differences in individual aspects by using a repeated-measures-ANOVA. We found a marginally significant difference for clarity, $F(1,14) = 3.65$, $p = .076$, and a clear difference in instructiveness, $F(1,15) = 26.3$, $p < 0.001$. For the comparison of *worked examples*, *completion assignments* and *open assignments*, we found significant differences in clarity, $F(2,32) = 14.3$, $p < .001$, and difficulty, $F(2,32) = 27.1$, $p < .001$. All pair-wise comparisons gave significant results too, so we conclude that worked examples were clearest and simplest, and that the open assignments were the least clear and the most difficult.

As the final part of the evaluation, we summarise remarks collected from the evaluation forms. We will focus here on the remarks collected in the Mathematica group. Some students expressed their general feeling about the course:

> *Working with Mathematica forms a good supplement to problem-solving on paper* (ID8).
>
> *After all it was fairly instructive, [....] A problem is, however, that while sitting behind the computer I can't think about physics problems too well.* (ID15).
>
> *It was fun to participate, but I did not find it very instructive ... The paper-and-pencil work groups are far!! more boring* (ID18).

Two students commented that the learning environment drew them into a passive attitude:

> *Solving the exercises tends to come down to using the "copy" and "paste" options of Mathematica. This did not contribute to the understanding of what really happened* (ID6).
>
> *[...] because of these worked examples you knew exactly what you had to do, so little initiative was required, whereas initiative should be important (you must be able to do it yourself, not to copy)* (ID31).

There were some positive remarks specifically about the visualisation facilities:

> *The benefit of the method is that the pictures give a good insight in what's going on. This may be helpful when you later come across a similar exercise. Pictures are easier to remember than formulas are* (ID7).
>
> *Working with Mathematica forms a good supplement to problem-solving on paper. The pictures give an insight in what you are working on* (ID8).

> *After all it was fairly instructive, especially the visualisation...* (ID15).

Finally, some students commented on problems they had with Mathematica:

> *Problems with Mathematica syntax cause a loss of time, especially during the first session* (ID15).
>
> *It is easy to lose you way, exercises were not hard but took a lot of time, because of irritating Mathematica* (ID30).
>
> *It was instructive with regard to Mathematica* (ID32).
>
> *The difficulty was more in Mathematica than in physics. The longer I used Mathematica, the faster I worked, and the more I could concentrate on physics problems* (ID33).

4.3 Discussion

The aim of the experiment was to assess the effectiveness of two forms of tutoring: the traditional one and a newly designed one. We had predicted a benefit for students taking the experimental course, especially on understanding of problem situations and choice of solution methods. We found no significant difference between the approaches. Moreover, among the more surprising outcomes of the experiment was that, when we corrected for prior-performance levels, we could not demonstrate a benefit of participating in the experiment. A plausible interpretation of this finding is that participants compensated for the extra effort they made in class by spending less time on their homework. However, as the controlled experiment we did was restricted to comparing the newly designed instruction with the traditional approach, we will restrict our further discussion to this comparison.

As a first point the quality and appropriateness of the tests used deserve attention. The reliability of the newly designed test proved to be dissatisfactory. As a consequence, we could not detect small differences in learning results. The regular test was rather reliable, but it had its focus on different abilities than the ones we were looking for. Even with these limited tests, probably we would have detected any major differences between both groups, however, so that we conclude that the effect of the experimental treatment has been limited.

An important factor explaining the lack of a gain for the newly designed instruction might be the high cognitive load in the computer course. We found strong evidence that the extraneous cognitive load in the computer condition is quite high. Both the closed evaluation items and the remarks made by the students suggest that students in the computer group were distracted from the physics content. Apart from the disruptive elements in the learning environment itself, there were external distractions, such as the availability of Internet browsers, that were absent in the traditional group.

When we examine the outcome of the evaluation in detail, we see that also the students' prior attitudes and expectations might have worked against the computer learning group. This approach to problem-solving was quite new for the students, and, also, the students were not yet fluent programmers. This is supported by informal observations made in the computer group. A first observation is that students initially generally disliked Mathematica. They had worked with a previous version of Mathematica (version 2.2.3) in a programming course, and most students disliked it. Moreover, several students expressed as their opinion that physics problem-solving is best done on paper, and that you cannot learn physics via a computer. This belief might be reinforced by the students'

experience with examinations, since in most examinations it is a requirement that the student can write out solutions fluently.

Although the learning environment we used provided more structure than the tasks the students had worked on in their programming course, students ran into many frustrating errors caused by typing mistakes and by more fundamental misunderstandings of programming principles. In addition to the students' mistakes, the version of Mathematica that we used (version 3.0.0.0) had some irritating bugs, and the program often had to be rebooted. As most students were quite new to programming, they were not very systematic in tracing the errors they made. Many even failed to diagnose that they had used non-matching parentheses in an expression. The problems were aggravated by the cryptic error messages Mathematica produces when it does not understand what it is supposed to do. Given this, the evaluation findings regarding the students in the computer group are not surprising.

The evaluation of the different components of the Mathematica learning environment indicates that the students found the visualisation facilities rather instructive. This impression is confirmed by some of the students' remarks on the evaluation form. Moreover, it is in line with observations by the experimenter, who had several discussions about common misconceptions that were triggered by visualisation outcomes. As an example, consider the following: one of the assignments was to draw a field plot for the field of a physical dipole, and compare this to the field of a mathematical dipole. One of the steps in the exercise was to zoom in on the physical dipole. Several students failed to understand why the plot essentially remained the same, and where they had to look for the changes. This situation provided a good starting point for a tutoring discussion. Although the students gave a high rating to the clarity of the visualisations, the quality of the plots could still be improved (for suggestions see Appendix B). Moreover, even though the students did not find the visualisations to be particularly difficult, it became clear that the students had to spend too much time on figuring out the details of the graphics commands.

The symbolic computation features were valued less positively, as indicated by the evaluation of the *compute gradient* facilities. Although the students judged these rather straightforward to use, they did not find it very instructive to compute gradients. A major cause of this problem could be that students fail to examine the computer-generated expressions critically, so that application of the symbolic computation facilities remains just a trick. This creates the impression that several students had not yet fully attained the formal reasoning level. This suggestion is strengthened by misconceptions we encountered, such as the one about the dipole field, mentioned earlier, and a student failing to understand why the method of image charges can only be used with virtual image-charges (i.e. with the position of the image charge being outside the region of interest).

The three major instructive elements in the experimental learning environment, namely worked examples, completion assignments and open assignments, were judged to be about equally instructive. As expected, the three elements were judged to become progressively more difficult. As indicated both by remarks on the evaluation form and by the experimenter's observations, the assignments could too often be solved by mindless copying of the worked examples. This may be because students spend so much of their time on the introductory, structured assignments. In any case, the opportunity to solve problems by just copying entire solutions must have worsened the students' attentiveness to computer-generated formulas.

To summarise, we found no significant differences between learning outcomes with the new approach and with the traditional approach. The students' remarks and our own

observations indicate that the new approach is potentially useful, however. This is especially true for the visualisation facilities. We found some problems with the instructional material. The most serious difficulties were related to navigation and to high extraneous cognitive load. Moreover, also the quality of the testing materials, and the correspondence between testing materials and course content deserve further attention.

5. Conclusions and recommendations

We have reported here on the development and evaluation of a learning environment for a first-year university course on electrostatics. Our goal was to support students who are moving toward the theoretical level-of-understanding in gaining an intuitive understanding of situations and solution methods. We have formulated the demands for such a learning environment, and then we have built a learning environment based on available software. The primary goals in designing the learning environment were to support visualisation and symbolic computation.

We tested a first version of the experimental course on a sample of first-year physics students. Results indicate that this approach to learning and problem-solving is quite new to the students and that it takes them a considerable amount of time to get used to the new approach. Moreover, students did not see how the abilities taught in the computer course were relevant to the final test, where they would have to solve the problems by hand anyway. If the students are to be won over to the approach, in the final test for the subject some items should be clearly related to the new approach. We found that the students in our experiment were more positive about the facilities for visualising solutions and situations than they are about the symbolic computation support. We interpret this to indicate that the students in the experiment have not fully attained the formal reasoning level. This is worrying because the students who participated were among the more proficient students in their cohort.

Although we could not demonstrate a significant learning gain over the old approach, we found that the computer course was successful in addressing misconceptions, in clarifying concepts that underlie solution methods and in supporting the construction of situation models. In all these learning episodes, discussions with the tutor played a central role, so we conclude that the role of the tutor in the experimental approach is as important as it is in the usual problem-solving workgroups.

On theoretical grounds we had chosen to give the learners full control of the learning environment and of their learning processes. This places a heavy cognitive load on the students, and so any additional extraneous cognitive load might harm learning. Regretfully, the experimental course contained several disruptive elements still. To explore the potential benefits of the new approach further, it is necessary to improve on the quality of the software and to refine the educational design of the course. The most urgent problems in the software are the following: the instability of the computer algebra software; the incomprehensible error messages, even about frequent errors such as mismatched parentheses; and the excessive programming effort for the student. The first problem may be solved in a next version or, alternatively, one may consider switching to an equivalent package such as *Maple*. The other problems we have to solve ourselves, as will other educators implementing similar courses. Improvements to the educational set-up of the course module should be aimed at increasing the students' reasoning about the physics background while they are studying worked examples and while they are solving completion assignments. In the current version, students could too easily solve the latter assignments by copying the examples (Appendix C suggests improvements for the present course).

We believe that, with these improvements, the revised course may provide a valuable supplement to practising electrostatics problem-solving by hand. Likewise, in other physics domains, such as mechanics, similar courses may help the students to gain an intuitive understanding of the abstract situations and methods they are working with.

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Appendix A Review of software

Mathematica

General description Mathematica is a symbolic and numerical computation language, with visualisation facilities. It is problem-solving tool rather than a learning environment per se.
Implementation the package consists of a separate kernel, to do all computations, and a hypertext shell to display input and output of text, 2D typeset formulas and – animated – graphics. Hyperlinks can point to other positions in the document as well at to other documents and URLs.
Impression The user is the master, once the command language is mastered. The user interface is powerful but not easy, and not too consistent as some seemingly equivalent commands give different results. A more restrictive user interface, with more informative error messages, could be easier to master. The programming language takes time to learn. You have to have a clear concept of goal: just playing around is not very funny with this package. Some features require too much programming, especially graphics commands. In general, the command language is farther from doing math than Maple's language is. The program has many bugs, even for a new major release.
Manufacturer Wolfram Research
Version 3.0.0
Platform Microsoft Windows 95/NT, various UNIX versions, Apple, NeXT
Similar functionality Maple, Derive

Maple

General description Maple is a symbolic and numerical computation language, with visualisation facilities. It is problem-solving tool rather than a learning environment per se.
Implementation the package consists of a separate kernel, to do all computations, and a hypertext shell to display input and output of text, 2D typeset formulas and – animated – graphics.
Impression The user is the master, once the command language is mastered. The user interface is consistent, but somewhat more limited that Mathematica's. The command language is close to doing math on paper. You have to have a clear concept of goal: just playing around is not very funny with this package. Much of the underlying code can be inspected and modified by the user.
Manufacturer Waterloo
Version V4
Platform Microsoft Windows 95/NT, various UNIX versions, Apple, NeXT
Similar functionality Mathematica, Derive

Matlab

General description Matlab is an integrated technical computing environment that combines numeric computation, visualisation, and simulations. In recent versions the kernel of the Maple package is included as a package to add symbolic computation facilities.
Implementation Matlab places a strong emphasis on high performance numerical code, with an accent on matrix and vector algebra. The command language resembles a programming language rather than conventional mathematics notation.
Impression Powerful and elegant command language, and an intelligent programming editor. A good graphical user interface as well, which allows the user to manipulate visualisations easily. As visualisation package, Matlab would make a good choice. As a symbolic problem solving tool, it has the disadvantage that formulas are presented as plain text, moreover, its syntax for symbolic functions is slightly less elegant than the original in Maple. On the other

hand, in Matlab symbolic functionality can also be addressed via an elegant calculator interface. A further strength of Matlab are its capabilities as a signal processing package, which may be used to combine with practicals. A final quality of Matlab, is that, combined with the associated package Simulink, it is a powerful tool for easily building and running simulations, which places it in the same category with Modellus and Labview.
Manufacturer MathWorks
Version 5.2
Platform Microsoft Windows 95, NT, Unix, Macintosh
Similar functionality Mathcad

Modellus

General description Modellus is a 2D visual simulation package that allows the user to define a physical situation in terms of equations. The equations can be explicitly time-dependent or they can be differential equations that develop in time. Though all kinds of interactions can be modelled in the equations, the visual elements that can be added are somewhat more limited: the objects themselves and different kinds of vectors can be displayed, alongside various xy graphs.
Implementation Simulations in Modellus are restricted to 2 dimensions. Modellus is mainly aimed at mechanics simulations, and it has no functionality for drawing fieldlines or equipotential surfaces. The equation editor supports differential equations, but not integrals.
Impression Conceptually this is it, moreover it is beautifully made, it connects pictural and the formal propositional representations. It is easy to learn, after only an hour I built a model of a forced, damped oscillator, including a dynamic visualisation with acceleration vectors etc.
Manufacturer Knowledge Revolution
Version 1.0
Platform Microsoft Windows 3.1
Similar functionality Labview, Simulink

SimQuest

General description SimQuest is an authoring system for creating learning environments that combine a computer simulation and learner support. The system is aimed at designing discovery learning environments.
Implementation SimQuest provides tools for creating simulations, for creating interfaces and for creating instructional support. SimQuest is an open system which implies that it can be used flexibly with all kinds of external simulations in addition. Up to now SimQuest has no features for working with formulas. The authoring system is still under development, but some courses that have been developed are stable. The design of the user-interface for learners is consistent, though a bit complex.
Impression Based on some sample courses, SimQuest allows attractive learning environments to be built. The core of a course is a simulation, defined by the teacher, where the learner can investigate the relations between a number of predefined parameters.
Manufacturer Servive Consortium, based at the University of Twente, The Netherlands
Version 1.1
Platform Microsoft Window 95
Similar functionality XYZet, Interactive Physics

XYZet

General description XYZ is a visual simulation package that allows the user to define or modify a 3D physical situation with both mechanical and electromagnetic interactions. The properties of the situation are all entered in a graphical interface. The time development of the

system is then simulated numerically. The visualisation encompasses the physical objects, forces and other vectors, field lines, and electromagnetic wave fronts

Implementation the package can handle many situations relevant to electrodynamics. The program runs smoothly. However, the design of the control panels is not very consistent, the field lines drawn are not distributed homogeneously in space, just one equipotential surface can be seen at a time, and there are some scaling problems with the magnitude of interactions.

Impression The tool provides a vivid, powerful intuitive learning environment. It was highly motivating to work with, also because the user is the owner of the problem, and moreover, it is easy to learn. The fundamental quantities and the physical laws are left implicit however. The link to physics theory is not made in the package itself however. For instance, with electromagnetic radiation, the wave front is beautifully visualised; it is not clear however why the wave front would move at the speed of light

Manufacturer IPN, Kiel, Germany. Distributed by Soft & Net

Version 1.04

Platform different Unix versions

Similar functionality Interactive Physics, SimQuest

Appendix B Sample from the experimental course

The first part of the section on dipole approximations.

Dipoolontwikkeling

Inleiding

```
Clear["Global`*"];
```

Op zeer grote afstand van een ladingsverdeling lijkt de potentiaal in zeergoede benadering op de potentiaal van een puntlading $\frac{Q}{4\pi\varepsilon_0 r}$. Waarbij Q de netto lading in de ladingsverdeling is. Maar wat nu als $Q=0$? In dat geval is de potentiaal in een ruwe benadering nul, maar we zoeken nu naar een betere benadering. Neem als voorbeeld de situatie van twee puntladingen q en min q met onderlinge afstand d. We kiezen de z-as door de twee ladingen, met de oorsprong midden tussen de twee ladingen.

```
vdipool[{x_, y_, z_}] :=
 Monopole[q, {0, 0, d/2}, {x, y, z}] + Monopole[-q, {0, 0, -d/2}, {x, y, z}]
vdipool[{x, y, z}]
```

$$\frac{q}{4\pi\sqrt{x^2+y^2+\left(-\frac{d}{2}-z\right)^2}\varepsilon_0}-\frac{q}{4\pi\sqrt{x^2+y^2+\left(\frac{d}{2}-z\right)^2}\varepsilon_0}$$

```
edipool[{x_, y_, z_}] = -Grad[vdipool[{x, y, z}], Cartesian[x, y, z]]
```

$$\frac{q\,x}{4\pi\left(x^2+y^2+\left(-\frac{d}{2}-z\right)^2\right)^{3/2}\varepsilon_0}-\frac{q\,x}{4\pi\left(x^2+y^2+\left(\frac{d}{2}-z\right)^2\right)^{3/2}\varepsilon_0},$$
$$\frac{q\,y}{4\pi\left(x^2+y^2+\left(-\frac{d}{2}-z\right)^2\right)^{3/2}\varepsilon_0}-\frac{q\,y}{4\pi\left(x^2+y^2+\left(\frac{d}{2}-z\right)^2\right)^{3/2}\varepsilon_0},$$
$$\frac{q\left(-\frac{d}{2}-z\right)}{4\pi\left(x^2+y^2+\left(-\frac{d}{2}-z\right)^2\right)^{3/2}\varepsilon_0}-\frac{q\left(\frac{d}{2}-z\right)}{4\pi\left(x^2+y^2+\left(\frac{d}{2}-z\right)^2\right)^{3/2}\varepsilon_0}\Big\}$$

We kunnen de equipotentiaallijnen en het elektrische veld samen afbeelden.

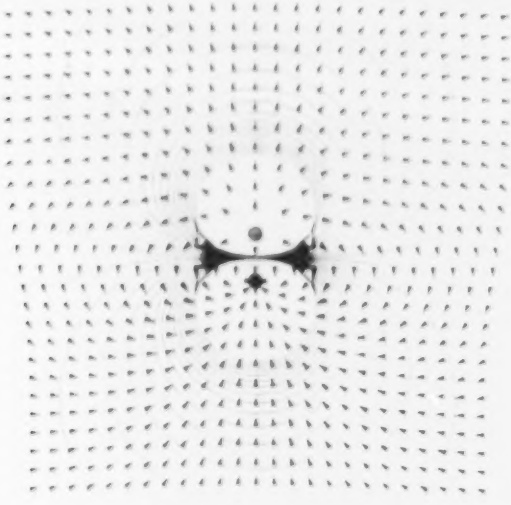
```
waarden = {q → 1, d → 1}
```

$$\{q \to 1,\ d \to 1\}$$

```
unite[{x_, y_, z_}] := UnitVec[edipool[{x, y, z}]]
plot1 =
 PlotVectorField[{unite[{x, 0, z}][[1]], unite[{x, 0, z}][[3]]} /. waarden,
  {x, -5, 5}, {z, -5, 5}, DisplayFunction → Identity, PlotPoints → 25];
plot2 = ContourPlot[vdipool[{x, 0, z}] /. waarden,
  {x, -5, 5}, {z, -5, 5}, ContourShading → False,
  PlotPoints → 30, Contours → 30, DisplayFunction → Identity];
```

```
lading1 = ParametricPlot[{0, d/2 /. waarden},
  {x, -2, 2}, PlotStyle → {RGBColor[0.5 + Sign[q]/2 /. waarden,
     0, 0.5 - Sign[q]/2 /. waarden], Thickness[0.03]},
  DisplayFunction → Identity];
lading2 = ParametricPlot[{0, -d/2 /. waarden},
  {x, -2, 2}, PlotStyle → {RGBColor[0.5 - Sign[q]/2 /. waarden,
     0, 0.5 + Sign[q]/2 /. waarden], Thickness[0.03]},
  DisplayFunction → Identity];
```

```
Show[plot1, plot2, lading1, lading2, DisplayFunction → $DisplayFunction]
```



```
- Graphics -
```

Opdracht: We zijn geinteresseerd in de potentiaal op grote afstand. Wat is 'grote' afstand in bovenstaande plot, relatief ten opzichte van wat? Bekijk de plot voor kleinere waarden van d (of voor grotere waarden van x en z).

De potentiaal van een dipool benaderd

```
Clear["Global`*"];
```

```
vdipool[{x_, y_, z_}] :=
 Monopole[q, {0, 0, d/2}, {x, y, z}] + Monopole[-q, {0, 0, -d/2}, {x, y, z}]
```

Het is handig op cilindercoordinaten over te gaan (waarom?)

```
vdipbolco[{r_, θ_, ϕ_}] :=
 vdipool[{r Cos[ϕ] Sin[θ], r Sin[ϕ] Sin[θ], r Cos[θ]}]
```

```
vdipbolco[{r, θ, ϕ}]
```

$$\frac{q}{4\pi\varepsilon_0\sqrt{\left(\frac{d}{2}-r\cos[\theta]\right)^2+r^2\cos[\phi]^2\sin[\theta]^2+r^2\sin[\theta]^2\sin[\phi]^2}}-\frac{q}{4\pi\varepsilon_0\sqrt{\left(\frac{d}{2}+r\cos[\theta]\right)^2+r^2\cos[\phi]^2\sin[\theta]^2+r^2\sin[\theta]^2\sin[\phi]^2}}$$

```
Simplify[%]
```

$$\frac{q\left(\frac{1}{\sqrt{\frac{d^2}{4}+r^2+dr\cos[\theta]}}-\frac{1}{\sqrt{\frac{d^2}{4}+r^2-dr\cos[\theta]}}\right)}{2\pi\varepsilon_0}$$

Als r veel groter wordt dan d dan krijgt deze formule de vorm 1/(r-een klein beetje)-1/(r+een klein beetje), naarmate r dus groter wordt zal het verschil tussen de twee termen kleiner worden. We willen daarom een reeksontwikkeling maken van de vorm: $V=c0+c1*\frac{1}{r}+c2*\frac{1}{r^2}+c3*\frac{1}{r^3}+\ldots$ Je herkent dit onmiddelijk als een Taylorreeksontwikkeling. De hogere orde termen vallen steeds sneller af met de afstand en dus zal de reeks voor grote afstand gedomineerd worden door de term met de laagste orde die ongelijk nul is. Als we een Taylor ontwikkeling in 1/r willen uitvoeren moeten we eerst een variabele inversr=1/r invoeren. We kunnen daarna een Taylorontwikkeling maken naar inversr.

```
% /. r → 1/inversr
```

$$\frac{q\left(\frac{1}{d^2\,\text{inversr}^2-4\,d\,\text{Cos}[\theta]\,\text{inversr}}-\frac{1}{d^2\,\text{inversr}^2+4\,d\,\text{Cos}[\theta]\,\text{inversr}}\right)}{2\pi\epsilon_0}$$

Het commando Series[f,{ x, x0, n }] maakt een reeksontwikkeling van functie f naar de variabele x, rondom punt x0. De hoogste orde term in de reeks is de term met $(x-x0)^n$. Als je een (Taylor)reeksontwikkeling maakt is de laatste term een restterm van de vorm $O[x^{n+1}]$. Dit is geen echte functie, de term geeft alleen aan dat er nog een rest is waarin x alleen voorkomt in de vorm x^{n+1} of hoger. (zie ook de help)

```
Series[%, {inversr, 0, 4}]
```

$$\frac{d\,q\,\text{Cos}[\theta]\,\text{inversr}^2}{4\pi\epsilon_0}+\frac{q\left(\frac{1}{4}\left(\frac{d^2}{4}-\frac{3}{4}d^2\,\text{Cos}[\theta]^2\right)+\frac{1}{4}\left(-\frac{d^2}{4}+\frac{3}{4}d^2\,\text{Cos}[\theta]^2\right)\right)\text{inversr}^3}{2\pi\epsilon_0}+\frac{q\left(-\frac{1}{4}d^3\,\text{Cos}[\theta]+\frac{5}{4}d\,\text{Cos}[\theta]\left(-\frac{d^2}{4}+\frac{3}{4}d^2\,\text{Cos}[\theta]^2\right)\right)\text{inversr}^4}{6\pi\epsilon_0}+O[\text{inversr}]^5$$

```
reeks = Simplify[% /. inversr → 1/r]
```

$$\frac{d\,q\,\text{Cos}[\theta]\left(\frac{1}{r}\right)^2}{4\pi\epsilon_0}+\frac{d^3\,q\,(3\,\text{Cos}[\theta]+5\,\text{Cos}[3\,\theta])\left(\frac{1}{r}\right)^4}{128\pi\epsilon_0}+O\left[\frac{1}{r}\right]^5$$

De eerste term ongelijk aan nul is een term met $1/r^2$. De volgende term is een term in $1/r^4$, de zogenaamde octopoolterm.

Je kunt de verschillende termen nu ieder een naam geven en afbeelden in een 3D plotje.

Als je de reeks wilt gebruiken in verdere berekeningen dan moet de restterm $O[x^{n+1}]$ verwijderd worden. Dit kan met het commando Normal[reeks]. Als je termen die aan bepaalde voorwaarden voldoen wilt selecteren uit een reeks dan kan dat met het commando Cases[reeks,zoekpatroon]. Zo levert bijvoorbeeld Cases[reeks, $\frac{1}{r^2}$] die termen op waarin r^2 voorkomt, de uitvoer van het commando Cases heeft de vorm van een lijst, om de accolades weg te werken selecteer je dus het element dat je nodig hebt ([1][bijvoorbeeld).

```
vdipterm[{r_, θ_}] = Cases[Normal[reeks], 1/r^2][[1]]
CylindricalPlot3D[vdipterm[{r, θ}] /. {q → 1, d → 1}, {r, 0.001, 3},
 {θ, 0, 2π}, BoxRatios → {1, 1, 1}]
```

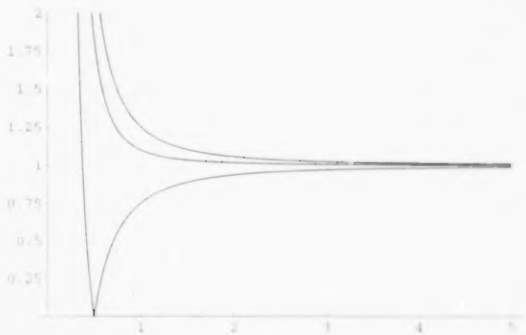
$$\frac{d\,q\,\text{Cos}[\theta]}{4\pi r^2\epsilon_0}$$

```
- Graphics3D -
```

CylindricalPlot3D plot een scalarfunctie op dezelfde manier als Plot3D, alleen nu met de argumenten van de functie in bolcoördinaten.

Opdracht Maak een plaatje van het relatieve verschil tussen a) de exacte uitdrukking voor de potentiaal en b) de dipoolbenadering als functie van de afstand r, doe dit voor een paar verschillende waarden van θ. Verklaar de vorm van de grafiek. Vanaf welke afstand zou je zeggen dat de dipoolbenadering een goede benadering is en wat voor criterium gebruik je daarvoor?

```
Plot[{vdipterm[{r, 0}] / vdipbolco[{r, 0, 0}] /. {q → 1, d → 1},
  vdipterm[{r, π / 4}] / vdipbolco[{r, π / 4, 0}] /. {q → 1, d → 1},
  vdipterm[{r, π / 3}] / vdipbolco[{r, π / 3, 0}] /. {q → 1, d → 1}}, {r, 0, 5}, PlotRange → {0, 2},
 PlotPoints → 200];
```



Appendix C Possible improvements to the course

Software and user interface

The other two improvements we can realise ourselves. The error messages could be implemented as a so-called 'package' that examines the input for frequently occurring beginner mistakes. A first reduction of programming effort could be easily achieved by offering a package that provides those graphics routines that do not contribute to the students' understanding of electrostatics, such as drawing the location of a point charge in a field plot. A further improvement could be the introduction of dedicated calculator interfaces for tasks such as solving Laplace's equation. This would relieve the student of keying in the commands, and moreover, it would provide a way to structure the output. Such a calculator interface could be a straightforward extension of the palettes already provided in Mathematica, such as the expression-input palette shown in the main text in Figure 1 on page 9.

A less urgent improvement to the software would be the introduction of continuous-field-line plots. Field plots in their current version are represented as a set of vectors drawn on the corners of a square grid. A plot with continuous field lines would be both more attractive and more readable (Figure 4). An algorithm for drawing such plots can be found in Tam (1997). However, it is not straightforward to implement such a routine in a generalised form.

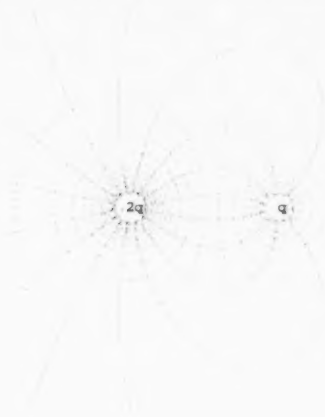


Figure 4 Example of a continuous field line plot, made with the algorithm described in Tam (1997).

Content and structure

A possible approach to stimulate studying the worked examples would be to pose interpretative questions about their physics content. The completion assignments could be revised in a way so that they require some more changes to be made by the students. Care should be taken, however, not to raise new programming problems in such a way.

The assignments could be modified to prevent students from mindless copying. It is intended that students could copy the structure of the example solution, but then several minor adaptations should be required to solve the problem so that the student has to work with the solution actively.

Reference

Tam, P.T. (1997) A physicist's guide to Mathematica, San Diego: Academic Press.

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